

Exercise Set 1Math 750Due Wed Jan. 20

1. Show that the union of the axes, $\{(x,y) \in \mathbb{R}^2 \mid xy=0\}$, is not a manifold.
2. Show that the cone $z^2 = x^2 + y^2$ in \mathbb{R}^3 is not a manifold.
3. Show that an atlas for S^n requires at least two charts.
4. Show that, if $E \rightarrow B$ is a covering with a countable number of leaves, then E is a manifold if and only if B is a manifold.
5. Given

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$$

let $h^+(x) \in S^n$ be the point of S^n on the ray

$$(1-t) \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ x_1 \\ \vdots \\ x_n \end{pmatrix}$$

with $t > 0$, and let $h^-(x) \in S^n$ be the point of S^n on the ray

$$(1-t) \begin{pmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ x_1 \\ \vdots \\ x_n \end{pmatrix}$$

with $t > 0$. Compute the "transition map" $h^- h^+ : \mathbb{R}^n - 0 \rightarrow \mathbb{R}^n - 0$. (Simplify your answer as far as possible.)

The End

Some good books.

Milnor, Topology from the Differentiable Viewpoint

Guillemin and Pollack, Differential Topology

Both of these assume manifolds come equipped with an embedding $M \subset \mathbb{R}^n$, for some n . This makes it hard to discuss manifolds

like

- projective spaces $\mathbb{R}P^n$, $\mathbb{C}P^n$, $\mathbb{H}P^n$
- Grassman manifolds (k -planes in \mathbb{R}^n)
- Stiefel manifolds (k -frames in \mathbb{R}^n)

More advanced:

Milnor, Characteristic Classes

Atiyah, K-Theory

Steenrod, The Topology of Fiber Bundles