Exercise Set #3 Math 750 W99 Due Friday Feb 5

1. Let $f: \mathbb{R}^3 \to \mathbb{R}^2$ by $f\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1^2 + x_2^2 + x_3^2 \\ x_1 x_2 x_3 \end{pmatrix}$.

(a) Compute Df

(b) Find all $x \in \mathbb{R}^3$ such that $Df(x)$ is onto.

2. Let $i: S^{n-1} \to \mathbb{R}^n$ be the inclusion map. Show that $Ti: TS^{n-1} \to T\mathbb{R}^n = \mathbb{R}^n \times \mathbb{R}^n$ is a homeomorphism from TS^{n-1} to $\{(x,y) \in \mathbb{R}^n \times \mathbb{R}^n \mid |x| = 1 \text{ and } x \cdot y = 0\}$.

3. Show that $f: \mathbb{R}^2 \to \mathbb{R}^3$ by $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ xy \\ y^2 \end{pmatrix}$ is an immersion except at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Describe the image $f(\mathbb{R}^2)$.

4. Let $f: \mathbb{R}^3 \to \mathbb{R}^4$ be $f\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1^2 - x_2^2 \\ x_1 x_2 \\ x_1 x_3 \\ x_2 x_3 \end{pmatrix}$.

(a) Show $f|_{\mathbb{R}^3 - L}$ is an immersion, where $L = \left\{ \begin{pmatrix} 0 \\ 0 \\ x \end{pmatrix} \middle| x \in \mathbb{R} \right\}$.

(b) If $i: S^2 \to \mathbb{R}^3$ is the inclusion, show that fi is an immersion.

(c) Show that fi induces a map $\bar{f}: \mathbb{R}P^2 \to \mathbb{R}^4$ which is an immersion.

(d) Show that $\bar{f}$ is an embedding. (This is the standard embedding of $\mathbb{R}P^2$ into $\mathbb{R}^4$.)

The End