Exercise Set 5 due Monday after Spring Break, 3/22/99

1. Define a diffeomorphism $D^n \xrightarrow{f} \mathbb{R}P^n - \mathbb{R}P^{n-1}$, where D^n is the open n-disk and $\mathbb{R}P^{n-1}$ is embedded in $\mathbb{R}P^n$ as $\{[x_0, \ldots, x_{n-1}, 0]\}$.

   Show that f extends to a map $\overline{D^n} \longrightarrow \mathbb{R}P^n$ which, when restricted to $S^{n-1} \subset \overline{D^n}$, gives the usual double covering $S^{n-1} \longrightarrow \mathbb{R}P^{n-1}$.

2. Let $E = \{(\ell, x) \in \mathbb{R}P^{n-1} \times \mathbb{R}^n \mid x \in \ell\}$, and embed $\mathbb{R}P^{n-1}$ into E as $\{(\ell, 0)\}$. Define a diffeomorphism

$$f : E \longrightarrow \mathbb{R}P^n - [0, \ldots, 0, 1]$$

   which is the usual embedding $\mathbb{R}P^{n-1} \longrightarrow \mathbb{R}P^n$ on $\{(\ell, 0)\}$.

   (This is an example of the Tubular Neighborhood Theorem: E is the normal bundle of $\mathbb{R}P^{n-1}$ in $\mathbb{R}P^n$ and $\mathbb{R}P^n - [0, \ldots, 0, 1]$ is an open nhood of $\mathbb{R}P^{n-1}$.)

<u>Note</u>: In (1) and (2), show that the map you define really is a diffeomorphism.

3. If $f : M^n \longrightarrow \mathbb{R}P^{n+k}$ is any continuous map, show that f is homotopic to a map g whose image is contained in $\mathbb{R}P^n$.