Test #1   Due Friday Feb. 12   by 4 PM in 1201 FAB.

Let M^m be a smooth manifold with smooth atlas $\mathcal{H} = \{h_\alpha\}_{\alpha \in A}$.

(1) Prove that M is parallelizable if and only if there exist smooth functions $\lambda_\alpha : U_\alpha \to Gl_n(\mathbb{R})$ such that

$$\lambda_\beta(h_{\beta\alpha}(x)) = Dh_{\beta\alpha}(x) \circ \lambda_\alpha(x)$$

for all $\alpha, \beta \in A$ and all $x \in U_{\beta\alpha}$.

(2) Prove $\mathbb{R}P^2$ is not parallelizable.

Hints: Consider the atlas $\{k_0, k_1, k_2\}$ and show that the $\lambda_0, \lambda_1, \lambda_2$ in (1) cannot exist by considering $\operatorname{sign}(\det \lambda_i)$.

(3) Prove that $\mathbb{R}P^3$ is parallelizable.

Hint: Do not use (1); instead think about the relationship of S^3 to $\mathbb{R}P^3$.

Instructions: Find elegant solutions. You may discuss the problem with friends, but you should write your solutions yourself.