Here is an easy proof that **C** is algebraically closed.

Proof. Suppose E is a division algebra over C of degree $n < \infty$. Since E is a division algebra, multiplication gives a map

$$(E \setminus 0) \times (E \setminus 0) \to (E \setminus 0).$$

Since **C** is central in *E* and multiplication in *E* is **C**-linear, it factors to give a map $\mu : \mathbf{CP}^{n-1} \times \mathbf{CP}^{n-1} \to \mathbf{CP}^{n-1}.$

Claim 1. $\mu^*(y) = 1 \otimes y + y \otimes 1$, where $y \in H^2 \mathbb{CP}^{n-1}$ is a generator.

Given this, we have

$$0 = \mu^*(y^n)$$

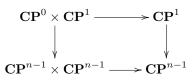
= $(1 \otimes y + y \otimes y)^n$
= $1 \otimes y^n + ny \otimes y^{n-1} + \dots + y^n \otimes 1$
= $ny \otimes y^{n-1} + \dots + ny^{n-1} \otimes y.$

from which it follows that n = 1.

Proof of the claim. choose an $e \in E \setminus (\mathbf{C} \cdot 1)$. Then, restricting the multiplication, we have

$$(\mathbf{C} \cdot 1 \setminus 0) \times (\mathbf{C} \cdot \{1, e\} \setminus 0) \to (\mathbf{C} \cdot \{1, e\} \setminus 0)$$

inducing



The top map is a homeomorphism, in fact, essentially the identity, and the vertical maps are well known, showing that $1 \otimes y$ occurs with coefficient 1 in $\mu^*(y)$. The other term is handled similarly.

Remark 2. This is the integral version of the argument, commonly given in introductory algebraic topology courses, that a real division algebra must have dimension a power of two. Oddly, I have not found this in the literature.