A RELATION IN THE STEENROD ALGEBRA
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Abstract. To make it as easy for the user as possible, we want to be able to specify a module over the Steenrod algebra by simply giving the Sq^{2^i}. Here, we point out a relation in the Steenrod algebra which gives an efficient way to compute all the Sq^i given only the Sq^{2^i}.

1. Introduction

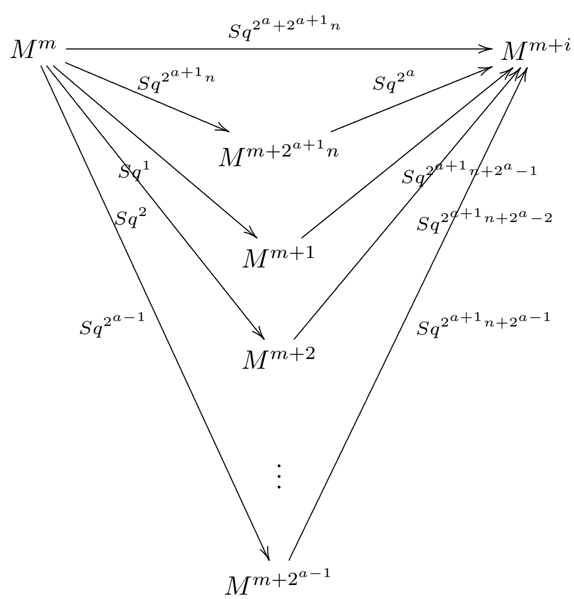
Let $\mathcal{A}$ be the mod 2 Steenrod algebra. Generalizing the relation $Sq^1 Sq^{2n} = Sq^{2n+1}$, we have the following relation in $\mathcal{A}$.

Proposition 1.

$$Sq^{2^{a+1}n+2^a} = Sq^{2^a} Sq^{2^{a+1}n} + \sum_{k=0}^{a-1} Sq^{2^{a+1}n+2^a-2^k} Sq^{2^k}$$

Proof. Expand $Sq^{2^a} Sq^{2^{a+1}n}$ by the Adem relations. The only terms which are nonzero are those occuring in the formula, by an elementary exercise in binomial coefficients. Specifically, the bits in the binary expansion of $2^a - 2j$ are all contained within the bits of the binary expansion of $2^{a+1}n - j - 1$ iff either $j = 0$ or $j = 2^k$ with $0 \leq k \leq a - 1$. □

Remark 2. *Therefore, we can compute Sq^i inductively by writing $i = 2^a + 2^{a+1}n$ and using the calculation in lower degrees inductively, together with the given action of the Sq^{2^k}.*



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References

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