Squaring operations in $\text{Ext}_{\mathcal{A}}(F_2, F_2)$
Robert R. Bruner

Values that are unknown are indicated by a dash. Machine calculated after feeding in hand calculations for the first few, including h_i, c_0, n.

s	t	x	0	1
1	1	h_0	h_1	h_0h_0
1	2	h_1	h_2	h_1h_1
1	4	h_2	h_3	h_2h_2
1	8	h_3	h_4	h_3h_3
1	16	h_4	h_5	h_4h_4
1	32	h_5	h_6	h_5h_5
1	64	h_6	h_7	h_6h_6

s	t	x	0	1	2	3
3	11	c_0	c_1	f_0	h_0e_0	h_3Ph_1
3	22	c_1	c_2	f_1	h_1e_1	h_1x
3	44	c_2	c_3	f_2	h_2e_2	h_2x_1

s	t	x	0	1	2	3	4
4	18	d_0	d_1	0	r	0	d_0d_0
4	21	e_0	e_1	x	t	m	d_0g
4	22	f_0	f_1	0	Y	h_3r	0
4	24	g	g_2	0	h_5Ph_2	$h_1h_5Ph_1$	gg
4	36	d_1	d_2	0	r_1	0	gg_2
4	37	p	p_1	h_0p'	h_1Q_3	h_2A''	0
4	42	e_1	e_2	x_1	t_1	m_1	$h_1x_{7,53}$
4	44	f_1	f_2	0	h_4Q_3	h_2t_1	0
4	48	g_2	g_3	0	0	h_5D_2	g_2g_2
4	65	D_3	$Sq^0(D_3)$	$x_{5,77}$	$x_{6,94}$	0	$h_1Sq^0(Q_2)$
4	72	d_2	d_3	0	r_2	0	0
4	73	p'	p'_1	$x_{5,93}$	−	−	$h_5x_{7,93}$
4	74	p_1	p_2	$h_1p'_1$	$h_2Sq^0(Q_3)$	$h_3A''_1$	0

s	t	x	0	1	2	3	4	5
5	14	Ph_1	h_2g	0	0	0	P^2h_2	$h_1P^2h_1$
5	16	Ph_2	0	h_4Ph_2	0	0	$h_0h_0i+h_1Pd_0$	$h_2P^2h_2$
5	36	n	n_1	r_1	−	−	−	0
5	42	x	x_1	0	$x_{7,57}+h_1x_{6,53}$	$h_0h_0x_{6,53}+h_1x_{7,53}$	−	$h_3x_{9,40}$
5	57	D_1	$Sq^0(D_1)$	−	−	−	0	0
5	67	H_1	$Sq^0(H_1)$	−	−	−	−	H_1H_1
5	72	n_1	n_2	r_2	−	−	−	0
5	72	Q_3	$Sq^0(Q_3)$	−	−	−	−	$h_4x_{9,117}$

s	t	x	0	1	2	3	4	5	6
6	36	r	r_1	0	0	0	–	0	rr
6	38	q	h_2Q_3	h_4G	0	0	$x_{10,35}$	$h_1x_{10,32}$	0
6	42	t	t_1	m_1	$h_0h_0x_{6,53}+h_1x_{7,53}$	0	0	0	$Ph_1x_{7,33}$
6	44	Y	h_4Q_3	h_2t_1	0	0	$x_{10,48}+e_0A'$	$h_4x_{10,27}$	0
6	56	C	C_1	0	$h_3x_{7,81}$	$h_3x_{8,83}$	–	–	0
6	60	G	G_1	–	–	–	–	0	0
6	64	D_2	0	–	–	–	–	–	D_2D_2
6	67	A	$h_0x_{5,80}+h_2x_{5,77}$	–	–	–	–	–	D_2A''
6	67	A'	A'_1	–	–	–	–	–	$A'A'$
6	70	A''	A''_1	$x_{7,124}+h_1x_{6,107}+h_3x_{6,97}$	–	–	–	–	0
6	72	r_1	r_2	0	0	0	$h_4x_{9,117}$	0	$h_2x_{11,189}$
6	77	$x_{6,47}$	$Sq^0(x_{6,47})$	–	–	–	–	–	$h_1x_{11,236}$
6	82	$x_{6,53}$	$Sq^0(x_{6,53})$	–	–	–	–	–	–

s	t	x	0	1	2	3	4	5	6	7
7	23	Pc_0	h_1Y	h_2m	–	0	0	$h_0x_{11,7}$	$h_0P^2e_0$	$h_3P^3h_1$
7	30	i	h_2C	0	gn	gr	0	0	$x_{13,14}$	ii
7	33	j	0	0	–	0	gm	$h_0h_0x_{10,19}+d_0gg$	ri	rPd_0
7	36	k	h_4C	–	–	–	$h_3x_{10,19}+nr$	rr	0	e_0z
7	39	l	$h_0x_{6,47}$	$h_3x_{7,33}$	–	–	–	–	gw	km
7	42	m	m_1	0	–	$h_3x_{9,40}$	0	0	tm	mm
7	53	B_1	$Sq^0(B_1)$	–	–	–	–	–	–	B_1B_1
7	55	B_2	$Sq^0(B_2)$	–	–	–	–	–	–	$h_5x_{13,29}$
7	64	Q_2	$Sq^0(Q_2)$	0	–	–	–	–	–	Q_2Q_2
7	67	B_3	$h_1x_{6,97}$	–	–	–	–	–	–	0
7	70	$x_{7,33}$	$x_{7,124}+h_1x_{6,107}+h_3x_{6,97}$	–	–	–	–	–	–	0
7	70	$x_{7,34}$	$h_3x_{6,97}$	–	–	–	–	–	–	0
7	73	$x_{7,40}$	$Sq^0(x_{7,40})$	–	–	–	–	–	–	$gx_{10,107}$
7	82	$x_{7,53}$	$Sq^0(x_{7,53})$	–	–	–	–	–	–	$c_1x_{11,155}$

Table (s = 8)

s	t	x	0	1	2	3	4	5	6	7	8
8	30	Pd_0	gd_1	0	gr	0	d_0d_0g	0	ii	0	$d_0P^2d_0$
8	33	Pe_0	0	—	gt	gm	d_0gg	$h_0h_0h_0x_{10,18}$	0	$e_0x_{11,7}$	$e_0P^2e_0$
8	54	$x_{8,20}$	$h_1h_6t + h_2x_{7,81}$	—	—	—	—	—	—	—	0
8	70	$x_{8,32}$	$x_{8,139} + h_2h_2x_{6,97}$	—	—	—	—	—	—	—	$h_1x_{15,110}$
8	70	$x_{8,33}$	$x_{8,139} + x_{8,140} + h_2h_2x_{6,97}$	—	—	—	—	—	—	—	0
8	76	$x_{8,43}$	$x_{8,176} + h_1h_3h_7Ph_1$	—	—	—	—	—	—	—	$g_2x_{12,58}$
8	77	$x_{8,46}$	$gSq^0(D_3)$	—	—	—	—	—	—	—	0
8	80	$x_{8,49}$	$h_2x_{7,164}$	—	—	—	—	—	—	—	0
8	82	$x_{8,51}$	$x_{8,200} + h_3x_{7,164}$	—	—	—	—	—	—	—	0

Table (s = 9)

s	t	x	0	1	2	3	4	5	6	7	8	9
9	26	P^2h_1	0	0	0	0	0	0	0	0	P^4h_2	$h_1P^4h_1$
9	28	P^2h_2	0	0	0	0	0	0	0	0	$h_0h_0P^2i + h_1P^3d_0$	$h_2P^4h_2$
9	48	u	$h_4x_{8,49}$	0	0	0	0	—	—	mw	$h_0x_{16,35}$	uu
9	51	v	0	—	—	0	—	—	—	—	—	uw
9	54	w	$h_2x_{8,83}$	—	—	—	—	—	—	0	$x_{8,20}w$	ww
9	69	$x_{9,29}$	$x_{9,145} + h_0x_{8,132} + h_2x_{8,119}$	—	—	—	—	—	—	—	—	$rx_{12,55}$
9	70	$x_{9,31}$	$h_0x_{8,136} + h_1x_{8,133} + h_6x_{8,43}$	—	—	—	—	—	—	—	—	$x_{9,31}x_{9,31}$
9	76	$x_{9,39}$	$x_{9,197}$	—	—	—	—	—	—	—	—	0
9	76	$x_{9,40}$	$h_2x_{8,161}$	—	—	—	—	—	—	—	—	$Ph_2x_{13,117}$

Table (s = 10)

s	t	x	0	1	2	3	4	5	6	7	8	9	10
10	51	z	0	—	0	—	—	—	—	—	—	—	$lx_{13,16}$
10	63	$x_{10,18}$	0	—	0	0	—	—	—	—	—	—	$x_{10,18}x_{10,18}$
10	64	$x_{10,19}$	$h_3x_{9,107}$	0	0	0	—	—	—	—	—	—	$ix_{13,46}$
10	66	$x_{10,22}$	0	0	0	—	0	—	—	—	—	—	$Pd_0x_{12,55}$
10	69	$x_{10,24}$	$h_4x_{9,111}$	—	—	—	—	—	—	—	—	—	$e_0x_{16,62}$
10	72	$x_{10,27}$	$h_4x_{9,117}$	—	—	—	—	—	—	—	—	—	$x_{10,27}x_{10,27}$
10	72	$x_{10,28}$	0	—	—	—	—	—	—	—	—	—	$x_{10,24}x_{10,34}$
10	74	$x_{10,32}$	$h_2x_{9,168}$	—	—	—	—	—	—	0	0	—	0
10	75	$x_{10,34}$	$x_{10,196} + x_{10,197} + h_1x_{9,178}$	—	0	—	0	0	—	0	—	—	$e_0x_{16,89}$
10	76	$x_{10,35}$	$h_3x_{9,168}$	0	0	—	0	0	0	0	—	0	0
10	76	$x_{10,36}$	0	—	—	—	0	0	0	0	—	—	0
10	79	$x_{10,40}$	0	0	—	—	0	0	—	—	—	—	0

Table (s = 11):

s	t	x	0	1	2	3	4	5	6	7	8	9	10	11
11	35	P^2c_0	0	0	0	0	0	0	—	0	0	$h_0x_{19,7}$	$h_0P^4e_0$	$h_3P^5h_1$
11	45	$x_{11,7}$	0	—	—	0	rw	kw	lz	d_0e_0z	$e_0x_{15,14}$	$ix_{13,14}+e_0x_{16,15}$	rP^2i	rP^3d_0
11	78	$x_{11,35}$	$h_3x_{10,187}$	—	—	—	—	—	—	—	—	—	—	$h_3x_{21,105}$

Table (s = 12):

s	t	x	0	1	2	3	4	5	6	7	8	9	10	11	12
12	42	P^2d_0	0	$Ph_1x_{7,33}$	0	—	0	iw	0	0	0	0	iP^2i	0	$d_0P^4d_0$
12	45	P^2e_0	0	—	nw	rw	kw	0	0	0	$h_0h_0h_0x_{18,20}$	0	$e_0x_{19,7}$	0	$e_0P^4e_0$

Table (s = 13):

s	t	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13
13	38	P^3h_1	0	0	0	0	0	0	—	—	0	—	0	0	P^6h_1	$h_1P^6h_1$
13	40	P^3h_2	0	0	0	0	0	—	0	—	0	0	0	0	$h_0h_0P^3i+h_1P^5d_0$	$h_2P^6h_1$
13	60	$x_{13,14}$	0	0	0	0	0	0	0	0	0	0	0	0	$h_0x_{24,34}$	$x_{13,14}x_{13,14}$
13	60	$x_{13,15}$	0	0	0	—	0	—	—	—	—	—	—	0	$h_2x_{24,30}$	$ux_{17,18}$
13	63	$x_{13,16}$	0	0	0	0	0	—	—	—	—	—	—	$vx_{15,14}$	—	$vx_{17,19}$
13	78	$x_{13,29}$	$h_1x_{12,194}$	—	—	—	—	—	—	—	—	—	—	—	—	$x_{13,29}x_{13,29}$
13	81	$x_{13,30}$	$h_1x_{12,213}$	—	—	—	—	—	—	—	—	—	—	—	—	$e_0x_{22,84}$
13	82	$x_{13,32}$	$g_2x_{9,102}$	—	—	—	—	—	—	—	—	—	—	—	—	$Ph_1x_{21,111}$

Table (s = 14, 15):

s	t	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
14	78	$x_{14,23}$	0	1	—	—	0	—	—	—	—	—	—	—	—	—	$x_{10,22}x_{18,23}$	$h_3P^7h_1$
15	47	P^3c_0	0	0	—	0	0	—	0	0	0	—	—	0	0	$h_0x_{27,7}$	$h_0P^6e_0$	iP^3i
15	54	P^2i	0	—	—	—	0	0	0	0	0	0	0	$lx_{19,7}$	0	0	$x_{29,14}$	rP^3i
15	57	$x_{15,6}$	0	0	0	0	grw	d_0mw	e_0mz	$vx_{13,16}$	0	0	0	0	$e_0x_{23,14}$	$x_{13,14}P^2i+e_0x_{24,15}$	rP^3i	rP^5d_0
15	69	$x_{15,14}$	0	0	0	0	0	0	0	0	mmw	ruw	rvz	$e_0wx_{13,16}$	$gzx_{13,16}$	$f_0x_{24,30}+d_0vx_{15,14}$	$P^3h_2x_{16,38}+d_0vx_{16,15}$	$vx_{21,17}$

Table 1 (columns 0–17)

s	t	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
16	54	$P^3 d_0$	0	—	—	—	—	—	—	0	$j x_{17,19}$	0	$l x_{19,7}$	0	$d_0 e_0 P^4 e_0$	0	$i P^3 i$	0	$d_0 P^6 d_0$	
16	57	$P^3 e_0$	0	0	gnw	grw	$d_0 mw$	0	0	$z x_{13,16}$	$l x_{17,19}$	0	0	$e_0 x_{23,14}$	$e_0 x_{24,15}$	$h_0 h_0 h_0 x_{26,18}$	0	$e_0 x_{27,7}$	$e_0 P^6 e_0$	
16	69	$x_{16,15}$	0	0	0	0	0	0	tmw	mmw	ruw	0	0	0	0	$P^3 h_2 x_{16,38}$	0	$z x_{21,17}$	$l x_{25,19}$	
17	50	$P^4 h_1$	0	0	—	0	0	0	0	0	0	0	0	—	0	0	0	0	$P^8 h_2$	$h_1 P^8 h_1$
17	52	$P^4 h_2$	0	0	—	—	0	—	—	—	—	—	—	—	—	0	0	0	$h_0 h_0 x_{31,5} + h_1 P^7 d_0$	$h_2 P^8 h_2$
17	72	$x_{17,18}$	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	$u x_{25,18}$
17	75	$x_{17,19}$	0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	$v x_{25,19}$

Table 2 (columns 0–12)

s	t	x	0	1	2	3	4	5	6	7	8	9	10	11	12
19	59	$P^4 c_0$	0	0	0	—	—	—	—	0	—	—	0	0	0
19	69	$x_{19,7}$	0	0	—	—	0	0	0	—	0	$d_0 rvw$	—	$d_0 lwz$	—
19	81	$x_{19,12}$	0	0	0	0	0	0	0	0	0	0	0	0	0
20	66	$P^4 d_0$	0	—	—	—	—	0	0	0	$wwww$	0	$d_0 kww$	—	0
20	69	$P^4 e_0$	0	—	—	0	0	—	0	0	www	$d_0 rvw$	$d_0 rww$	$d_0 lwz$	0
21	62	$P^5 h_1$	0	0	0	0	0	0	0	0	0	0	0	0	0
21	64	$P^5 h_1$	0	—	0	—	0	0	0	0	—	0	0	0	0
23	71	$P^5 c_0$	0	—	0	0	0	—	0	0	—	—	0	0	0
23	78	$P^3 i$	0	—	—	—	—	0	—	—	—	—	—	0	—
23	81	$x_{23,6}$	0	—	—	—	—	0	0	—	—	—	—	0	—
24	78	$P^5 d_0$	0	—	—	—	0	—	—	—	—	—	0	0	—
24	81	$P^5 e_0$	0	—	—	0	0	0	—	—	—	—	—	—	—
25	74	$P^6 h_1$	0	0	—	0	0	0	0	0	—	—	—	0	0
25	76	$P^6 h_1$	0	0	0	0	0	0	0	0	0	—	0	0	0

Table 3 (columns 13–25)

s	t	x	13	14	15	16	17	18	19	20	21	22	23	24	25
19	59	$P^4 c_0$	—	—	0	0	$h_0 x_{35,7}$	$h_0 P^8 e_0$	$h_3 P^9 h_1$	0	0	0	0	0	0
19	69	$x_{19,7}$	—	—	0	0	—	—	$x_{38,14}$	0	0	0	0	0	0
19	81	$x_{19,12}$	0	0	—	$x_{35,51}$	$x_{36,43} + f_0 x_{32,32}$	$x_{37,41}$	$x_{38,37}$	$x_{39,12}$	$x_{40,13}$	0	0	0	0
20	66	$P^4 d_0$	—	—	—	—	—	$x_{38,10}$	0	$x_{40,7}$	0	0	0	0	0
20	69	$P^4 e_0$	—	—	—	0	0	0	$x_{39,12}$	$x_{40,13}$	0	0	0	0	0
21	62	$P^5 h_1$	0	0	0	0	0	0	0	$P^{10} h_2$	$h_1 P^{10} h_1$	0	0	0	0
21	64	$P^5 h_1$	—	—	—	—	—	0	0	$h_0 h_0 P^4 i + h_1 P^9 d_0$	$h_2 P^{10} h_2$	0	0	0	0
23	71	$P^5 c_0$	—	—	—	—	—	—	—	0	$h_0 x_{43,7}$	$h_0 P^{10} e_0$	$h_3 P^{11} h_1$	0	0
23	78	$P^3 i$	—	—	—	—	—	—	—	—	0	—	$x_{46,10}$	0	0
23	81	$x_{23,6}$	—	—	—	—	—	—	—	—	0	$x_{46,10}$	$x_{46,14}$	$x_{48,7}$	0
24	78	$P^5 d_0$	—	—	—	—	—	—	—	—	—	—	0	$x_{47,12}$	0
24	81	$P^5 e_0$	—	—	0	0	—	—	—	—	—	—	$x_{47,12}$	$x_{48,13}$	0
25	74	$P^6 h_1$	0	0	0	0	0	—	—	0	0	0	0	$P^{12} h_2$	$h_1 P^{12} h_1$
25	76	$P^6 h_1$	0	—	0	0	0	0	0	0	0	0	0	$h_0 h_0 x_{47,5} + h_1 P^{11} d_0$	$h_2 P^{12} h_2$